KIAS-P01047
hep-th/0110270

Comments on D-branes on general group manifolds

Yoji Michishita *

School of Physics, Korea Institute for Advanced Study
207-43, Cheongryangri, Dongdaemun, Seoul, 130-012, Korea

October, 2001

Abstract

We investigate D-branes with maximal symmetry on general group manifolds in terms of boundary states and effective actions. We show that boundary states with an suitable Wilson line form boundary states of the other types of D-branes, extending the known fact in SU(2) case. We also show that fluctuation mass spectrum around D-brane solutions of the effective action agrees with that of boundary CFT.

*michishi@kias.re.kr

1 Introduction

D-branes on flat space has been investigated extensively in the last several years, and various properties on D-branes on curved space has also been clarified gradually.

For extending the analysis of D-branes to curved space, group manifolds are good examples since CFTs on them i.e. WZW models are exactly solvable and geometrical meaning is clearer than many other abstract CFTs. Recently D-branes on SU(2) have been studied extensively. One of the interesting phenomena is that D2-branes can be regarded as bound states of D0-branes[1, 2].

In this letter we investigate D-branes with maximal symmetry on general group manifolds in terms of boundary states and effective actions extending the work of SU(2) case. We will consider maximally symmetric D-branes given by Cardy boundary states. In particular we consider formation of D-branes by gauge field condensation.

This letter is organized as follows. In section 2 we briefly review D-brane on general group manifolds in terms of WZW models, and comment on the exponent of the characters in the case of D0-branes. In section 3 we extend the boundary state analysis given in [3] to general group manifolds. We show that when we turn on a Wilson line on a bunch of D-branes it forms other types of D-branes. Then we comment on supersymmetry. In section 4 we consider effective actions given in [2, 4] , solutions of the equation of motion and fluctuation mass spectrum. We show that the spectrum agrees with that of CFT. We also comment on tachyon potentials.

2 Maximally symmetric D-branes on general group manifolds

When we consider D-branes in WZW models we must impose some boundary condition which relates the left moving current J^a to the right moving current $\bar{J}^a$. Here we consider the following condition which preserves the symmetry of the current algebra (in terms of open strings).

$$J = r\bar{J}r^{-1}, \tag{1}$$

where r is an element of the group G, $J = J^a T^a$ and T^a are generators of G. The action of r on $\bar{J}$ is an inner automorphism. In addition to this one can consider outer automorphisms, though we do not treat them in this paper.

It is known that the D-brane with this boundary condition is wrapped on $C_h \cdot r$, where C_h is the conjugacy class of h [5]. Furthermore h is 'quantized' by quantum consistency.

The building blocks of boundary states are called Ishibashi states [6], which are constructed for each primary operator. We denote the Ishibashi state for the boundary condition (1) constructed on the primary field belonging to the representation with the highest weight μ (We always denote representations of Lie algebra by their highest weights.) by $|\mu, r\rangle\rangle$ Then $|\mu, r\rangle\rangle$ is defined as follows.

$$|\mu, r\rangle\rangle \equiv \sum_n |\mu, n\rangle_L \otimes e^{2\pi i \theta^a \bar{J}_0{}^a} U \, |\mu, n\rangle_R \,, \tag{2}$$

where $|\mu, n\rangle$ are orthonormal bases of the module constructed on the representation μ, and θ^a is defined by $r = e^{2\pi i \theta^a T^a}$. U is an operator which satisfy $U^{-1} J_n U = -J^\dagger_{-n}$.

Then we can construct boundary states consistent with the Cardy condition by taking appropriate linear combination of $|\mu, r\rangle\rangle$. In the case of the diagonal modular invariant partition function they are given as follows[7] .

$$|\lambda, r\rangle = \frac{S_\lambda{}^\mu}{\sqrt{S_\mu{}^0}} \, |\mu, r\rangle\rangle, \tag{3}$$

where $S_\lambda{}^\mu$ is the modular transformation matrix of characters of current algebra (See e.g. [8]):

$$\chi_\lambda(\zeta/\tau, -1/\tau) = S_\lambda{}^\mu q^{\frac{k}{2}|\zeta|^2} \chi_\mu(\zeta, \tau). \tag{4}$$

Characters $\chi_\lambda(\zeta, \tau)$ are defined as follows.

$$\chi_\lambda(\zeta, \tau) = \mathrm{tr}_\lambda \left[q^{L_0 - \frac{c}{24}} \exp\left(-2\pi i \zeta \cdot H\right) \right], \quad q = e^{2\pi i \tau}. \tag{5}$$

H^i are generators of the Cartan subalgebra.

$S_\lambda{}^\mu$ has the following properties.

$$S^{-1} = S^\dagger, \quad {}^t S = S, \quad (S_\lambda{}^\mu)^* = S_{\lambda^*}^{\ \mu} = S_\lambda{}^{\mu^*}, \tag{6}$$

$$S_\lambda{}^\mu = \widetilde{\chi}_\mu \left(\frac{-2\pi i}{k+g} (\lambda + \rho) \right) S_\lambda{}^0, \tag{7}$$

where $\widetilde{\chi}_\mu$ is the character of ordinary Lie algebras. λ^* is the conjugate representation of λ. k, g and ρ are level of the current algebra, the dual Coxeter number and the Weyl vector respectively.

We can read off the open string spectrum by computing the cylinder amplitude using these boundary states and modular transforming to the open string picture:

$$\begin{aligned}\langle\lambda, r'|\, \tilde{q}^{\frac{1}{2}(L_0+\bar{L}_0)-\frac{c}{24}}\, |\mu, r\rangle &= \frac{(S^\dagger)_\sigma{}^\lambda S_\mu{}^\sigma}{S_\sigma{}^0}\chi_\sigma(-\zeta, -1/\tau) \\ &= \frac{S_{\lambda^*}{}^\sigma S_\mu{}^\sigma (S^\dagger)_\sigma{}^\omega}{S_\sigma{}^0} q^{\frac{k}{2}|\zeta|^2}\chi_\omega(\tau\zeta, \tau) \\ &= N_{\lambda^*\mu}^{\omega} q^{\frac{k}{2}|\zeta|^2}\chi_\omega(\tau\zeta,\tau),\end{aligned} \tag{8}$$

where we used $\langle\langle\lambda, r'|\tilde{q}^{\frac{1}{2}(L_0+\bar{L}_0)-\frac{c}{24}}\,|\mu, r\rangle\rangle = \langle\langle\lambda, 1|\tilde{q}^{L_0-\frac{c}{24}}e^{-2\pi i\theta'^a J_0^a}e^{2\pi i\theta^a J_0^a}\,|\mu, 1\rangle\rangle = \chi_\mu(-\zeta, -1/\tau)$, and the Verlinde formula [9]. ζ^a is defined by $e^{2\pi i\zeta^a T^a} = e^{-2\pi i\theta'^a T^a}e^{2\pi i\theta^a T^a}$. Here we assumed, without loss of generality, ζ^a has nonzero value only when the index a is along the direction of Cartan subalgebra. Notice that (8) corresponds to open strings from $|\mu, r\rangle$ to $|\lambda, r'\rangle$. For open strings with the opposite orientation λ, r' and μ, r are exchanged.

$|0, r\rangle$ corresponds to D0-brane. For D0-branes conjugacy classes must be trivial. Therefore the position of $|0, r\rangle$ in G is r.

In general power of q in the partition function is α' times mass squared of modes of strings. Hence we can expect that the power of the factor $q^{\frac{k}{2}|\zeta|^2}$ in (8) is equal to α' times the square of the tension times the length of geodesic connecting r and r'. We will show that it is correct.

The metric on G can be read off from the worldsheet action of the WZW model. We denote the parameter of the geodesic by s. $s = 0$ and $s = 1$ correspond to r and r' respectively. Then the length L is

$$L = \int_0^1 ds\sqrt{\left(-\frac{k\alpha'}{2}\right)\mathrm{tr}[g^{-1}\partial_s g g^{-1}\partial_s g]}. \tag{9}$$

The geodesic can be derived by minimizing L:

$$\partial_s(\partial_s g g^{-1}) = 0. \tag{10}$$

The solution of this equation is

$$g(s) = re^{-2\pi i\zeta s}. \tag{11}$$

Therefore,

$$L = 2\pi\sqrt{\frac{k\alpha'}{2}}|\zeta|^2. \tag{12}$$

Then $\alpha'(L/2\pi\alpha')^2 = \frac{k}{2}|\zeta|^2$, as expected.

3 Wilson lines and boundary states

In [3] it is shown that, in the SU(2) case, boundary states with the following Wilson line operator satisfies the boundary condition (1) again.

$$\mathrm{P}\mathrm{tr}\exp\left(-\frac{i}{k}\int_0^{2\pi} d\sigma J^a(\sigma)M^a\right), \quad [M^a, M^b] = if^{abc}M^c. \tag{13}$$

This fact is also true for general group manifolds. Indeed, the argument given in [3] can be applied to the case of general group manifolds straightforwardly. In addition to this, it is shown that in the large k limit we can get D2-brane boundary states by acting the above Wilson line operator on boundary state of D0-branes.

We will show this is also true in the case of general group by extending the calculation given in [3].

First we determine how the Wilson line operator acts on each Ishibashi state. Since the Wilson line operator does not change boundary condition as mentioned above, and its action closes in each module, the result must be proportional to the Ishibashi state itself:

$$\mathrm{P}\mathrm{tr}\exp\left(-\frac{i}{k}\int_0^{2\pi} d\sigma J^a(\sigma)M^a\right)|\lambda, r\rangle\rangle = c(\mu,\lambda,k)\,|\lambda, r\rangle\rangle, \tag{14}$$

where M^a are representation matrices of μ. To extract $c(\mu,\lambda,k)$ we can concentrate only on the primary state part of the Ishibashi state. In the large k limit, we can remove the path ordering of the Wilson line.

$$\mathrm{tr}\exp\left(-\frac{2\pi i}{k}J_0^a M^a\right)|\lambda, r\rangle\rangle = \sum_{\mu'}\langle\mu,\mu'|\exp\left(-\frac{2\pi i}{k}J_0^a M^a\right)|\mu,\mu'\rangle\otimes|\lambda, r\rangle\rangle, \tag{15}$$

where μ' is a weight of μ. The operator $J_0^a M^a$ in the exponent can be diagonalized as follows.

$$\begin{aligned} J_0^a M^a &= \frac{1}{2}[(J_0^a + M^a)^2 - (J_0^a)^2 - (M^a)^2] \\ &= \frac{1}{2}[(\nu,\nu+2\rho) - (\lambda,\lambda+2\rho) - (\mu,\mu+2\rho)], \end{aligned} \tag{16}$$

where we used the quadratic Casimir operator Q_λ for a representation λ is equal to $(\lambda,\lambda+2\rho)$. ν is the representation coming from the tensor product of λ and μ. Then $c(\mu,\lambda,k)$ can be read off as follows.

$$c(\mu,\lambda,k) = \sum_\nu \mathcal{N}_{\mu\lambda}^{\nu}\exp\left(-\frac{\pi i}{k}[(\nu,\nu+2\rho) - (\lambda,\lambda+2\rho) - (\mu,\mu+2\rho)]\right), \tag{17}$$

where $\mathcal{N}_{\mu\lambda}^{\nu}$ is the tensor product coefficient, i.e. the multiplicity of the representation ν in the decomposition of the tensor product $\mu \otimes \lambda$. This is given by the following formula derived by the character method.

$$\mathcal{N}_{\mu\lambda}^{\nu} = \sum_{\substack{\mu' \in \{\text{weights of } \mu\} \\ w \in \text{Weyl group }, w(\lambda+\mu'+\rho)-\rho=\nu\in\{\text{dominant weights}\}}} \epsilon(w) \text{mult}_{\mu}(\mu'). \tag{18}$$

where $\epsilon(w)$ is the sign of w and $\text{mult}_{\mu}(\mu')$ is the multiplicity of the weight μ' in the representation μ.

We assume that only $|\lambda_i - \mu_i| \gg 1$ and $|\lambda_i| \gg 1$ contribute, where λ_i and μ_i are the Dynkin labels of λ and μ. Then, only the unit element of the Weyl group contribute to the sum of (18), because λ, and $(\lambda + \mu' + \rho)$ by the above assumption, are in the fundamental chamber, and $w(\lambda + \mu' + \rho)$, and $w(\lambda + \mu' + \rho) - \rho$ by the above assumption again, are outside the fundamental chamber for nontrivial w. Therefore,

$$\mathcal{N}_{\mu\lambda}^{\nu} \sim \begin{cases} \text{mult}_{\mu}(\mu') & \text{for} \quad \nu = \lambda +^{\exists} \mu', \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

Now we can compute $c(\mu, \lambda, k)$:

$$\begin{aligned} c(\mu,\lambda,k) &\sim \sum_{\mu'} \text{mult}_{\mu}(\mu') \exp\left(-\frac{\pi i}{k}[(\lambda+\mu',\lambda+\mu'+2\rho)-(\lambda,\lambda+2\rho)-(\mu,\mu+2\rho)]\right) \\ &= \sum_{\mu'} \text{mult}_{\mu}(\mu') \exp\left(-\frac{\pi i}{k}[2(\lambda+\rho,\mu')-(\mu,\mu+2\rho)+(\mu',\mu')]\right) \\ &\sim \sum_{\mu'} \text{mult}_{\mu}(\mu') \exp\left(-\frac{2\pi i}{k+g}(\lambda+\rho,\mu')\right) \\ &= \widetilde{\chi}_{\mu}\left(-\frac{2\pi i}{k+g}(\lambda+\rho)\right) \\ &= \frac{S_{\lambda}{}^{\mu}}{S_{\lambda}{}^{0}}. \end{aligned} \tag{20}$$

Then we can determine the action of the Wilson line:

$$\begin{aligned} \text{Ptr} \exp\left(\frac{i}{k}\int_0^{2\pi} d\sigma J^a(\sigma) M^a\right)|\nu, r\rangle &= \text{Ptr} \exp\left(\frac{i}{k}\int_0^{2\pi} d\sigma J^a(\sigma) M^a\right)\frac{S_{\nu}{}^{\lambda}}{\sqrt{S_{\lambda}{}^{0}}}|\lambda, r\rangle\rangle \\ &= \frac{S_{\nu}{}^{\lambda}}{\sqrt{S_{\lambda}{}^{0}}}\frac{S_{\lambda}{}^{\mu}}{S_{\lambda}{}^{0}}|\lambda, r\rangle\rangle \\ &= \frac{S_{\nu}{}^{\lambda}}{\sqrt{S_{\lambda}{}^{0}}}\frac{S_{\mu}{}^{\lambda}}{S_{\lambda}{}^{0}}(S^{\dagger})_{\lambda}{}^{\xi} S_{\xi}{}^{\pi}|\pi, r\rangle\rangle \end{aligned}$$

$$
\begin{aligned}
&= N^{\xi}_{\nu\mu}\frac{S_{\xi}{}^{\pi}}{\sqrt{S_{\pi}{}^{0}}}\,|\pi,r\rangle\rangle \\
&= N^{\xi}_{\nu\mu}\,|\xi,r\rangle\,. \qquad (21)
\end{aligned}
$$

This shows that by the action of the Wilson line D-branes $|\nu,r\rangle$ form other types of D-branes. The multiplicity of $|\xi,r\rangle$ is $N^{\xi}_{\nu\mu}$. In particular, $|\nu,r\rangle$ can be regarded as a bound state of dim ν D0-branes.

Finally we comment on supersymmetry. In supersymmetric WZW models the current J^a can be written as a sum of fermion part and remaining part independent of the fermion:

$$
J^a = j^a - \frac{i}{2k}f^a{}_{bc} : \psi^b\psi^c : . \qquad (22)
$$

If the index a is in the Cartan subalgebra, then

$$
J^i = j^i + \frac{1}{k}\sum_{\alpha>0}\alpha^i : \psi^{\alpha}\psi^{-\alpha} :, \qquad (23)
$$

where αs are positive roots. We can bosonize ψ^{α} as follows.

$$
\psi^{\pm\alpha} = \frac{\sqrt{2k}}{|\alpha|}e^{\pm i\phi_{\alpha}}, \quad \phi_{\alpha}(z)\phi_{\alpha}(w) \sim -\ln(z-w), \qquad (24)
$$

where we omit cocycle factors. Then,

$$
J^i = j^i + 2i\sum_{\alpha>0}\frac{\alpha_i}{|\alpha|^2}\partial\phi_{\alpha}. \qquad (25)
$$

Spacetime supercharges $Q_{\epsilon_{\alpha_1}\epsilon_{\alpha_2}\cdots}$ are written by spin operators $\exp(\frac{i}{2}\sum_{\alpha>0}\epsilon_{\alpha}\phi_{\alpha})$, where $\epsilon_{\alpha} = \pm 1$. When the boundary state $|\lambda,1\rangle$($\times$ other part from the remaining part of the spacetime) preserves the following combination of left moving and right moving supersymmetries,

$$
(Q_{\epsilon_{\alpha_1}\epsilon_{\alpha_2}\cdots} + \Lambda_{\epsilon_{\alpha_1}\epsilon_{\alpha_2}\cdots}^{\epsilon'_{\alpha_1}\epsilon'_{\alpha_2}\cdots}\bar{Q}_{\epsilon'_{\alpha_1}\epsilon'_{\alpha_2}\cdots})\,|\lambda,1\rangle = 0, \qquad (26)
$$

then, noting $|\lambda,r\rangle = \exp(-i\theta^i J^i_0)\,|\lambda,1\rangle$, we can see $|\lambda,r\rangle$ preserves the following supersymmetry.

$$
\left(Q_{\epsilon_{\alpha_1}\epsilon_{\alpha_2}\cdots} + \exp\left(i\sum_{\alpha>0}\epsilon_{\alpha}\frac{\theta^i\alpha_i}{|\alpha|}\right)\Lambda_{\epsilon_{\alpha_1}\epsilon_{\alpha_2}\cdots}^{\epsilon'_{\alpha_1}\epsilon'_{\alpha_2}\cdots}\bar{Q}_{\epsilon'_{\alpha_1}\epsilon'_{\alpha_2}\cdots}\right)|\lambda,r\rangle = 0. \qquad (27)
$$

As is pointed out in [3], the sum of $|\lambda,1\rangle$ and $|\lambda,r\rangle$ does not preserve any supersymmetry in the case of SU(2). This is because the factor $\exp(i\sum_{\alpha>0}\epsilon_{\alpha}\frac{\theta^i\alpha_i}{|\alpha|})$ becomes $\exp(i\epsilon\theta/2)$ in this case, and if this factor is equal to 1, then $r=1$. However, if we consider group with the rank more than 2, we may put $\exp(i\sum_{\alpha>0}\epsilon_{\alpha}\frac{\theta^i\alpha_i}{|\alpha|}) = 1$ without putting $r=1$. For example, in the case of SU(3) $\theta = 2\omega_1 + \omega_2$ is such a choice, where ω_i are the fundamental weights. This fact shows that, in particular, two or more D0-branes put on different positions may be supersymmetric when we embed WZW models with the rank more than 2 in superstring theory.

4 Classical solutions of the effective action and mass spectrum of fluctuation around them

We assume that the background is (flat time direction)$\times G \times$other part which we do not consider. The time direction is introduced only for measuring masses. We consider only the state $J^a_{-1}|0\rangle$ in the case of bosonic string (and $\psi^a_{-1/2}|0\rangle$ in the case of superstring). The effective action has been determined in [2, 4] up to overall normalization:

$$S = \mathrm{tr}\left(\frac{2\alpha'}{k}(\partial_t B_a)^2 + \frac{1}{4k^2}[B_a, B_b]^2 - \frac{i}{3k^2} f^{abc} B_a [B_b, B_c]\right). \tag{28}$$

Here we do not distinguish upper and lower adjoint indices. The trace is taken over Chan-Paton factor. B_a are Hermitian matrices. The power of k for each term can be understood by noting that open string metric used for summation over adjoint indices contain k [2]. α' in front of the time derivative comes from dimensional analysis.

The equation of motion for static configurations is

$$[B_b, [B_a, B_b] - i f^{abc} B_c] = 0. \tag{29}$$

In [2, 4] it is pointed out that this equation has the solution

$$B_a = S_a, \quad [S_a, S_b] = i f^{abc} S_c, \tag{30}$$

and it represents the D-brane corresponding to the boundary state $|\lambda, 1\rangle$ if S_a is the representation matrix of λ. Direct sum of this type of solutions is also a solution and it represents the configuration that various D-branes are present simultaneously. In addition to these,

$$B_a = \begin{pmatrix} c_a & 0 \\ 0 & S^a \end{pmatrix}, \tag{31}$$

where c_a is a constant, is also a solution and in [10] it is claimed that in the SU(2) case this represents a D2-brane and a D0-brane. c_a corresponds to the position of the D0-brane.

Here we consider the following solution.

$$B_a = S_a + c_a \cdot 1. \tag{32}$$

We show that this solution corresponds to the boundary state $|\lambda, r\rangle$ by computing mass spectrum of fluctuations around it and comparing with that of CFT. c_a corresponds to r. Similar calculations for SU(2) have been done in [10] and [11, 12, 13] in different context.

Let us consider the direct sum of two such solutions:

$$B_a^0 = \begin{pmatrix} S^a + c_a \cdot 1 & 0 \\ 0 & S'^a + c'_a \cdot 1 \end{pmatrix}, \tag{33}$$

where S^a and S'^a are generators of representations λ and μ respectively. The low lying CFT spectrum of the configuration corresponding to this solution i.e. $|\lambda, r\rangle$ and $|\mu, r'\rangle$ can be read off from (8). The results are shown in table 1 and 2.

	$\vert\text{primary}\rangle$	
	representation	mass squared
open strings on $\vert\lambda, r\rangle$	$\sum_\sigma N^\sigma_{\lambda\lambda^*}\sigma$	$\frac{1}{\alpha'}\left(\frac{Q_\sigma}{2(k+g)} + \alpha' m^2\right)$
open strings on $\vert\mu, r'\rangle$	$\sum_\sigma N^\sigma_{\mu\mu^*}\sigma$	$\frac{1}{\alpha'}\left(\frac{Q_\sigma}{2(k+g)} + \alpha' m^2\right)$
open strings between $\vert\lambda, r\rangle$ and $\vert\mu, r'\rangle$	$\sum_\sigma N^\sigma_{\lambda\mu^*}\sigma$ $+\sum_\sigma N^\sigma_{\mu\lambda^*}\sigma$	$\frac{1}{\alpha'}\left(\frac{Q_\sigma}{2(k+g)} - \zeta_i \sigma'_i + \frac{k}{2}\vert\zeta\vert^2 + \alpha' m^2\right)$ σ' : weights of σ

Table 1: representations and mass spectrum of the states $|\text{primary}\rangle$

	$J^a_{-1}\vert\text{primary}\rangle$ or $\psi^a_{-1/2}\vert\text{primary}\rangle$	
	representation	mass squared
open strings on $\vert\lambda, r\rangle$	$\sum_\sigma N^\sigma_{\lambda\lambda^*}\sigma\otimes(\text{adjoint})$	$\frac{1}{\alpha'}\left(\frac{Q_\sigma}{2(k+g)}\right)$
open strings on $\vert\mu, r'\rangle$	$\sum_\sigma N^\sigma_{\mu\mu^*}\sigma\otimes(\text{adjoint})$	$\frac{1}{\alpha'}\left(\frac{Q_\sigma}{2(k+g)}\right)$
open strings between $\vert\lambda, r\rangle$ and $\vert\mu, r'\rangle$	$\sum_{\sigma,\nu} N^\sigma_{\lambda\mu^*}N^\nu_{\sigma,\text{adj.}}\nu$ $+\sum_{\sigma,\nu} N^\sigma_{\mu\lambda^*}N^\nu_{\sigma,\text{adj.}}\nu$	$\frac{1}{\alpha'}\left(\frac{Q_\sigma}{2(k+g)} - \zeta_i \nu'_i + \frac{k}{2}\vert\zeta\vert^2\right)$ ν' : weights of ν

Table 2: representations and mass spectrum of the states $J^a_{-1}|\text{primary}\rangle$ (bosonic string) or $\psi^a_{-1/2}|\text{primary}\rangle$ (superstring)

Quadratic Casimir operators in the table 1 and 2 come from dimensions of primary operators. m^2 in the table 1 is the contribution of zero point energy. It is given by $m^2 = -\frac{1}{\alpha'}$ in the case of bosonic D-branes and $m^2 = -\frac{1}{2\alpha'}$ in the case of non-BPS D-branes in superstring theory.

Here we note the open strings from $|\mu, r'\rangle$ to $|\lambda, r\rangle$ give complex conjugate representations of the open strings from $|\lambda, r\rangle$ to $|\mu, r'\rangle$ and, however, they give the identical mass spectrum since the sign of both weights and ζ flip.

If we plug $B_a = B_a^0 + \delta B_a$ into the action and calculate the part quadratic in the fluctuation δB_a, then we obtain

$$\begin{aligned}
\delta^2 S &= \frac{\alpha'}{k}\mathrm{tr}\left((\partial_t \delta B_a)^2\right) \\
&\quad + \frac{1}{k^2}\mathrm{tr}\left(\frac{1}{2}[B_a^0, \delta B_b]^2 - \frac{1}{2}[B_a^0, \delta B_a]^2 + [B_a^0, B_b^0][\delta B_a, \delta B_b] - i f^{abc} B_a^0 [\delta B_b^0, \delta B_c^0]\right). \quad (34)
\end{aligned}$$

We take the following as gauge fixing terms [11].

$$\frac{1}{k^2}\mathrm{tr}\left(\frac{1}{2}[B_a^0, \delta B^a]^2 + [B_a^0, b][B_a^0, c]\right). \quad (35)$$

b and c are antighost and ghost respectively. Since we are interested in only physical spectrum, we drop the ghost term henceforth. The first term of (35) cancels the second term in the second trace of (34). If we put δB_a as follows,

$$\delta B_a = \begin{pmatrix} D_a & E_a \\ E_a^\dagger & F_a \end{pmatrix}, \quad (36)$$

and use the explicit form of B_a^0, we obtain

$$\begin{aligned}
\delta^2 S &= \frac{\alpha'}{k}\mathrm{tr}\left((\partial_t D_a)^2 + \frac{1}{2\alpha' k}[S_a, D_b]^2\right) \\
&\quad + \frac{\alpha'}{k}\mathrm{tr}\left((\partial_t D_a)^2 + \frac{1}{2\alpha' k}[S'_a, F_b]^2\right) \\
&\quad \frac{2\alpha'}{k}\mathrm{tr}\left(\partial_t E_a \partial_t E_a^\dagger\right) + \frac{1}{k^2}\mathrm{tr}\Big((S'_a E_b^\dagger - E_b^\dagger S_a)(S_a E_b - E_b S'_a) \\
&\quad -2(c_a - c'_a) E_b^\dagger (S_a E_b - E_b S'_a) + 2i f^{abc}(c_a - c'_a) E_b^\dagger E_c \\
&\quad -(c_a - c'_a)^2 E_b^\dagger E_b\Big). \quad (37)
\end{aligned}$$

D_a, F_a and E_a correspond to strings on $|\lambda, r\rangle$, on $|\mu, r'\rangle$ and betweeen $|\lambda, r\rangle$ and $|\mu, r'\rangle$, respectively. Let us consider the part containing D_a. We define matrices N_a as follows.

$$\begin{aligned}
[S_a, D_b]_i^j &= (S_a)_i^k (D_b)_k^j - (D_b)_i^k (S_a)_k^j \\
&= [(S_a)_i^k \delta_l^j - \delta_i^k (S_a^*)_l^j](D_b)_k^l \\
&\equiv (N_a D_b)_i^j. \quad (38)
\end{aligned}$$

Indices i, j, k, l run from 1 to $\dim\lambda$. We can expand vectors with these indices by the basis of $(\dim\lambda)$-dimensional representation λ of G. Then we can think of N_a as generators of the tensor product representation $\lambda \otimes \lambda^*$. The mass term of D_a can be rewritten as follows.

$$\begin{aligned}
\mathrm{tr}\left(\frac{1}{2\alpha' k}[S_a, D_b]^2\right) &= \mathrm{tr}\left(-\frac{1}{2\alpha' k} D_b [S_a, [S_a, D_b]]\right) \\
&= -\frac{1}{2\alpha' k}(D_b)_j^i ((N_a)^2 D_b)_i^j. \quad (39)
\end{aligned}$$

$(N_a)^2$ is the quadratic Casimir operator. We can diagonalize this operator by decomposing $\lambda\otimes\lambda^*$ into $\sum_\sigma \mathcal{N}^\sigma_{\lambda\lambda^*}\sigma$. Therefore $\dim G\cdot\mathcal{N}^\sigma_{\lambda\lambda^*}\cdot\dim\sigma$ modes have mass squared $\frac{Q_\sigma}{2\alpha' k}$. (The factor $\dim G$ comes from the index b of D_b.) This spectrum agrees with that given by CFT (table 2) in large k limit, since $k+g\sim k$ and $\mathcal{N}^\sigma_{\lambda\lambda^*}\sim N^\sigma_{\lambda\lambda^*}$. Similarly the mass spectrum of F_b agrees with the CFT.

Next we compute the mass spectrum of $(E_a)^{i'}_i$. We can expand vectors with the index i and i' by the bases of λ and μ^* respectively. If we define L_a by $(S_aE_b - E_bS'_a)^{i'}_i \equiv (L_aE_b)^{i'}_i$, L_a can be regarded as the generators of $\lambda\otimes\mu^*$. Then,

$$\begin{aligned}\delta^2 S &= \frac{2\alpha'}{k}\Bigg(\partial_t(E^\dagger_a)^i_{i'}\partial_t(E_a)^{i'}_i - \frac{1}{2\alpha' k}\Big[(E^\dagger_b)^i_{i'}((L_a)^2E_b)^{i'}_i \\ &\quad +2(c_a-c'_a)(E^\dagger_b)^i_{i'}((L_a)E_b)^{i'}_i - 2if^{abc}(c_a-c'_a)(E^\dagger_b)^i_{i'}(E_c)^{i'}_i \\ &\quad +(c_a-c'_a)^2(E^\dagger_b)^i_{i'}(E_b)^{i'}_i\Big]\Bigg). \end{aligned}\tag{40}$$

Noting that $if^{abc}\equiv(T^b_{\text{adj.}})_{ac}$ is generators of adjoint representation, the second line of (40) can be written by $J^a\equiv L_a + T^a_{\text{adj.}}$:

$$2(c_a-c'_a)(E^\dagger_b)^i_{i'}((L_a)E_b)^{i'}_i - 2if^{abc}(c_a-c'_a)(E^\dagger_b)^i_{i'}(E_c)^{i'}_i = 2(c_a-c'_a)(E^\dagger_b)^i_{i'}(J_a)^{ci'j}_{bij'}(E_c)^{j'}_j. \tag{41}$$

Without loss of generality, we can assume that $c_a - c'_a$ has nonzero value only when the index a is along the direction of Cartan subalgebra.

Now we can read off the mass spectrum. The second term of (40) can be diagonalized in the same way as in the case of D_b: $\dim G\cdot\mathcal{N}^\sigma_{\lambda\mu^*}\cdot\dim\sigma$ modes have the eigenvalue $\frac{Q_\sigma}{2\alpha' k}$. The third and fourth term can be diagonalized by decomposing $\lambda\otimes\mu^*\otimes(\text{adjoint})$ into irreducible representations. J_a gives their weights because of the above assumption. Therefore $2\cdot\mathcal{N}^\sigma_{\lambda\mu^*}$ modes (The factor 2 is present because each component of E_b is complex.) have the eigenvalue $2(c_i-c'_i)\nu'_i$, where ν is a weight of $\sigma\otimes(\text{adjoint})$. The fifth term is already diagonalized. Then the total eigenvalues $\frac{1}{\alpha'}[\frac{Q_\sigma}{2k}+\frac{1}{k}(c_i-c'_i)\nu'_i+\frac{1}{2k}(c_i-c'_i)^2]$ is exactly the same as that obtained by CFT (table 2) in the large k limit, if we identify $(c_i-c'_i)$ with $-k\zeta_i$.

Next let us include the tachyon field in the effective action. The quadratic part of the action of the tachyon field is as follows.

$$S_T = \mathrm{tr}\left(\alpha'(\partial_t T)^2 + \frac{1}{2k}[B_a,T]^2 - \alpha' m^2T^2\right). \tag{42}$$

m^2T^2 is the mass term of T. The term $[B_a,T]$ comes from the kinetic term "$(D_aT)^2$". We consider the fluctuation around the solution (33) and $T=0$. We put the fluctuation of T as follows.

$$\delta T = \begin{pmatrix} X & Y \\ Y^\dagger & Z \end{pmatrix}. \tag{43}$$

Then the quadratic part in δT is

$$\begin{aligned}
\delta^2 S_T &= \alpha' \mathrm{tr}\left((\partial_t X)^2 + \frac{1}{2\alpha' k}[S_a, X]^2 - m^2 X^2\right) \\
&\quad \alpha' \mathrm{tr}\left((\partial_t Z)^2 + \frac{1}{2\alpha' k}[S'_a, Z]^2 - m^2 Z^2\right) \\
&\quad +2\alpha' \mathrm{tr}(\partial_t Y^\dagger \partial_t Y) + \frac{1}{k}\mathrm{tr}\Big((S'_a Y^\dagger - Y^\dagger S_a)(S_a Y - Y S'_a) \\
&\quad -2(c_a - c'_a) Y^\dagger (S_a Y - Y S'_a) - (c_a - c'_a)^2 Y^\dagger Y\Big) - \mathrm{tr}(2\alpha' m^2 Y^\dagger Y) \\
&= \alpha' \left((\partial_t X)^i_j (\partial_t X)^j_i - \frac{1}{2\alpha' k} X^i_j ((N_a)^2 X)^j_i - m^2 X^i_j X^j_i\right) \\
&\quad +\alpha' \left((\partial_t Z)^{i'}_{j'} (\partial_t Z)^{j'}_{i'} - \frac{1}{2\alpha' k} Z^{i'}_{j'} ((N'_a)^2 Z)^{j'}_{i'} - m^2 Z^{i'}_{j'} Z^{j'}_{i'}\right) \\
&\quad +2\alpha' \Bigg((\partial_t Y^\dagger)^i_{i'} (\partial_t Y)^{i'}_i - \frac{1}{2\alpha' k} (Y^\dagger)^i_{i'} ((L_a)^2 Y)^{i'}_i \\
&\quad -\frac{1}{\alpha' k}(c_a - c'_a)(Y^\dagger)^i_{i'} (L_a Y)^{i'}_i - \frac{1}{2\alpha' k}(c_a - c'_a)^2 (Y^\dagger)^i_{i'} Y^{i'}_i - m^2 (Y^\dagger)^i_{i'} Y^{i'}_i\Bigg).
\end{aligned} \tag{44}$$

We can read off the mass spectrum in the same way as in the previous case:

$$\begin{aligned}
X &: N^\sigma_{\lambda\lambda^*} \text{modes with mass squared } \frac{1}{\alpha'}\left(\frac{Q_\sigma}{2k} + \alpha' m^2\right) \\
Z &: N^\sigma_{\mu\mu^*} \text{modes with mass squared } \frac{1}{\alpha'}\left(\frac{Q_\sigma}{2k} + \alpha' m^2\right) \\
Y &: 2N^\sigma_{\lambda\mu^*} \text{modes with mass squared } \frac{1}{\alpha'}\left(\frac{Q_\sigma}{2k} + \frac{1}{k}(c_i - c'_i)\sigma'_i + \frac{1}{2k}(c_i - c'_i)^2 + \alpha' m^2\right)
\end{aligned} \tag{45}$$

These spectra agree with those given by CFT (table 1).

Finally we comment the form of the tachyon potential.

If we consider only the state $|0\rangle$, the calculation of the potential by BSFT [14] can be done trivially.

$$V(T) = \begin{cases} \mathrm{tr}(e^T(1+T)) & \text{for bosonic D-branes,} \\ \mathrm{tr}(e^{-\frac{1}{4}T^\dagger T}) + \mathrm{tr}(e^{-\frac{1}{4}TT^\dagger}) & \text{for brane-antibrane pairs,} \\ \mathrm{tr}(e^{-\frac{1}{4}T^2}) & \text{for non-BPS D-branes.} \end{cases} \tag{46}$$

Note that the tachyon fields in BSFT are different from T in the previous discussion. They are related by some nonlocal field redefinition. In the case of large k limit of SU(2) including nontrivial primary operators, see [15].

The above results are true not only on group manifolds, but also on general backgrounds. Furthermore they are exact regardless of the value of k. We can discuss vanishing of D-branes in the same way as on flat background.

Acknowledgments

I would like to thank M. Nozaki and S.-J. Sin for helpful correspondence and conversation.

References

[1] C. Bachas, M. Douglas and C. Schweigert, "Flux stabilization of D-branes", `hep-th/0003037`, *JHEP* **0005** (2000) 048

[2] A. Yu. Alekseev, A. Recknagel and V. Schomerus, "Brane Dynamics in Background Fluxes and Non-commutative Geometry", `hep-th/0003187`, *JHEP* **0005** (2000) 010

[3] Y. Hikida, M. Nozaki and Y. Sugawara, "Formation of spherical D2-brane from multiple D0-branes", `hep-th/0101211`

[4] S. Fredenhagen and V. Schomerus, "Branes on group manifolds, gluon condensates, and twisted K-theory", `hep-th/0012164`,*JHEP* **0104** (2001) 007

[5] A. Yu. Alekseev and V. Schomerus, "D-brane in the WZW model", `hep-th/9812193`, *Phys. Rev.* **D60** (1999) 061901

[6] N. Ishibashi, "The boundary and crosscap states in conformal field theories", *Mod. Phys. Lett.* **A4** (1989) 251 ; N. Ishibashi and T. Onogi, "Conformal field theories on surfaces with boundaries and crosscaps", *Mod. Phys. Lett.* **A4** (1989) 161

[7] J. L. Cardy, "Boundary conditions, fusion rules and Verlinde formula", *Nucl. Phys.* **B324** (1989) 581

[8] P. di Francesco, P. Mathieu and D. Sénéchal, "Conformal Field Theory", Springer

[9] E. Verlinde, "Fusion rules and modular transformations in 2D conformal field theory", *Nucl. Phys.* **B300** (1988) 360

[10] K. Hashimoto and K. Krasnov, "D-brane solutions in non-commutative gauge theory on fuzzy sphere", `hep-th/0101145`, *Phys. Rev.* **D64** (2001) 046007

[11] S. Iso, Y. Kimura, K. Tanaka and K. Wakatsuki, "Noncommutative gauge theory on fuzzy shpere from matrix model", hep-th/0101102, *NP* **B604** (2001) 121

[12] S. Bal and H. Takata, "Interaction between two fuzzy spheres", hep-th/0108002

[13] D. P. Jatkar, G. Mandal, S. R. Wadia and K. P. Yogendran, "Matrix dynamics of fuzzy spheres", hep-th/0110172

[14] A. A. Gerasimov and S. L. Shatashvili, "On exact tachyon potential in open string field theory", *JHEP* **0010** (2000) 034, hep-th/0009103 ; D. Kutasov, M. Mariño and G. Moore, "Some exact results on tachyon condensation in string field theory", *JHEP* **0010** (2000) 045, hep-th/0009148 ; D. Kutasov, M. Mariño and G. Moore, "Remarks on tachyon condensation in superstring field theory", hep-th/0010108

[15] H. Ita and Y. Oz, "String field theory and the fuzzy sphere", hep-th/0106187